Semiconductor Physics

9.1 Introduction

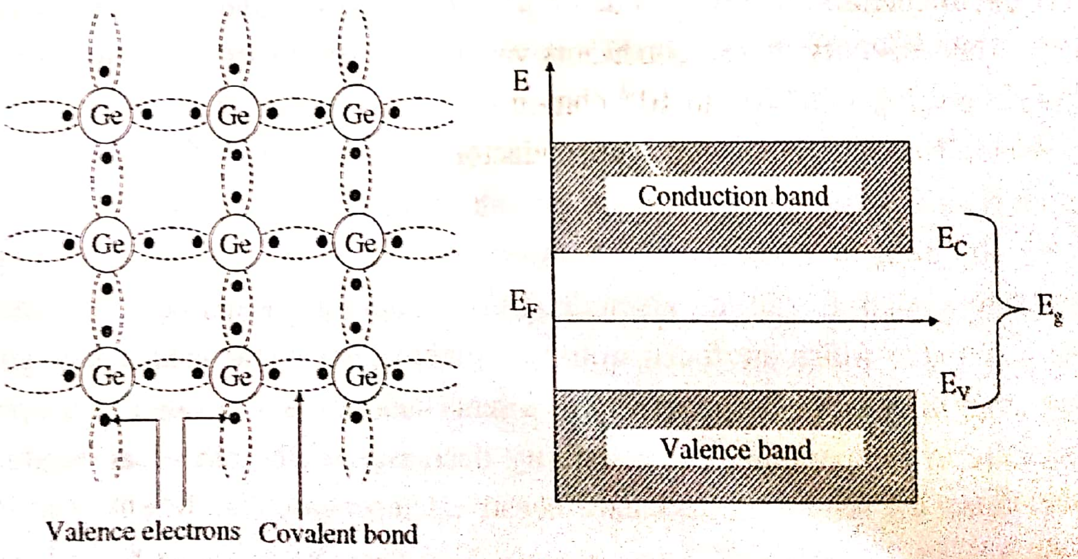
Semiconductors are materials, whose electronic properties are intermediate between those of good conductors and insulators. These intermediate properties are determined by the crystal structure, bonding characteristics, electronic energy bands and also by the fact that unlike metals, a semiconductor has both positive and negative carriers of electricity. The resistivity of semiconductors varies from 10^{-5} to 10^{+4} ohm-m as compared to the values ranging from 10^{-8} to 10^{-6} ohm-m for conductors and from 10^{7} to 10^{8} ohm-m for insulators. There are elemental semiconductors such as germanium (Ge) and silicon (Si) which belong to the group IV of the periodic table and have resistivity of about 0.6 and 1.5×10^{3} ohm-m respectively. Besides these, there are certain compound semiconductors such as gallium arsenide (GaAs), indium phosphide (InP), cadmium sulphide (CdS) etc., which are found from the combinations of the elements of group III and V or group II and IV. The resistivity of a semiconductor is a function of temperature. With increase of temperature, the resistivity decreases so the electrical conductivity increases. Hence the semiconductors have negative temperature co-efficient of resistance.

The band gap of semiconductors varies from 0.2 to 2.5 eV, which is quite small as compared to that of insulators. The band gap of a typical insulator such as diamond is about 6 eV. The conductivity and the effective band gaps of semiconductors can be changed by doping the impurities which strongly affect their electronic and optical properties. These properties determine the wavelength of radiation which can be emitted or absorbed by the semiconductor and hence help to construct devices such as light emitting diodes (LEDs) and lasers. The semiconductors are classified as,

1. Intrinsic (or) Pure semiconductors
2. Extrinsic (or) Impure semiconductors

9.2 Intrinsic Semiconductors

The pure form of a semiconductor is called an intrinsic semiconductor. A pure crystal of silicon or germanium is an intrinsic semiconductor. The electrical conductivity of this type of semiconductor is solely determined by thermally generated carriers. In order to understand the electrical conduction in an intrinsic semiconductor, let us consider the case of germanium. The atomic number of germanium is 32, so it has 32 electrons; 2 in the first orbit, 8 in the second orbit, 18 in the third orbit and remaining four in the outermost orbit. Thus the germanium atom has four valence electrons i.e., it is a tetravalent element. Each of the four valence electrons in a germanium atom is shared by the valence electrons of four adjacent germanium atoms and makes four electron pairs as shown in the Fig. 9.1(a). These types of electron pairs are known as covalent bonds which provide the bonding force between neighbouring atoms. In this way, the atoms behave as if their outermost orbits were complete with 8 electrons. Thus, free electrons are available to conduct a current through a germanium crystal. A pure germanium crystal is an insulator at absolute temperature.



(a) Bond structure (b) Energy band diagram

Fig. 9.1 Bond structure and energy band diagram of germanium crystal

However, with increase of temperature, a few covalent bonds are broken. When a ovalent bond is broken, one electron becomes free and leaves the atom and becomes a ositive ion known as hole. The hole acts as a positively charged particle, having a harge equal to that of an electron but of opposite sign. Since a hole is a strong centre of ttraction for the electron, the hole attracts an electron from the neighbouring atom to fill it. Thus the hole is now shifted to another place from where the electron has migrated. The newly created hole is filled up by another neighbouring electron and so on. Thus a hole once created moves about in the crystal in a random manner, just like the free electron. On applying an electric field, the holes move in a direction opposite to that of the valence electrons. It constitutes a hole current. It must be noted that in an intrinsic semiconductor, the number of electrons and holes are equal and they are less. Hence the current produced in a semiconductor is not adequate for any useful work. The energy band diagram of intrinsic semiconductor is shown in Fig. 9.1(b). The Fermi level E_F is at the middle of valence and conduction bands. If E_V and E_c are the energy levels of valence and conduction bands respectively, then the energy gap is,

$$E_g = E_c - E_v$$

and the Fermi energy is,

$$E_F = \frac{E_c + E_v}{2}$$

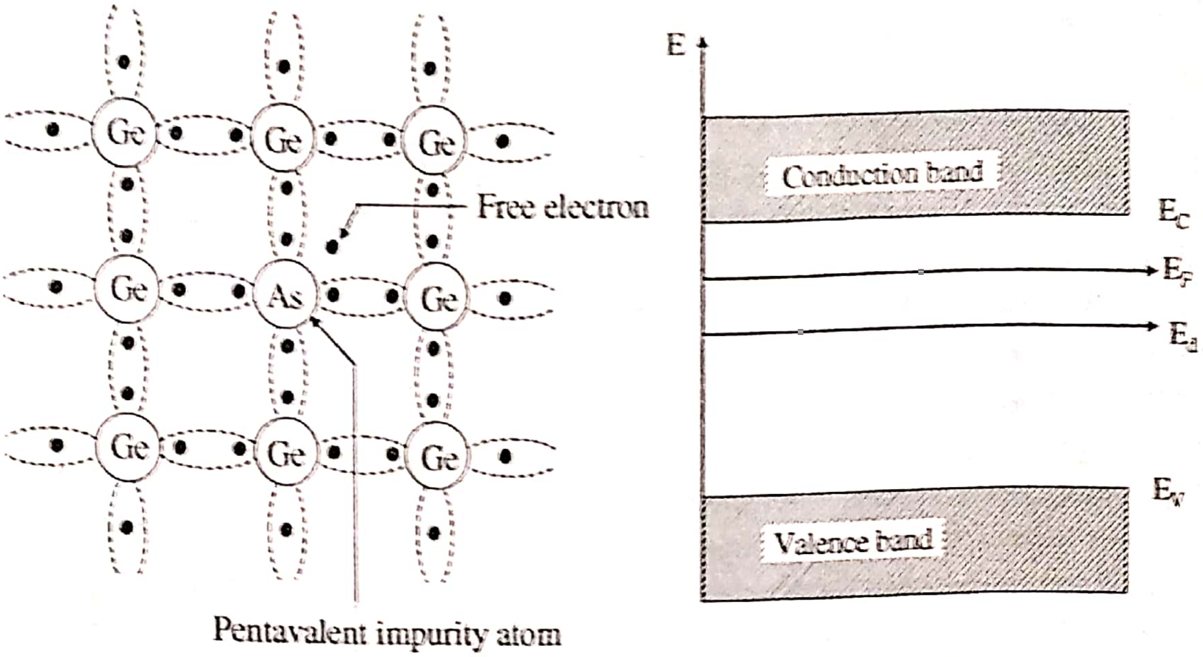
9.3 Extrinsic Semiconductors

If a small amount of impurity is added to a pure semiconductor, it significantly increases the conducting properties. The process of adding the impurities to a semiconductor is known as doping. A semiconductor doped with impurities is called extrinsic semiconductor. Depending upon the type of impurity added to pure semiconductors (like Ge or Si) the extrinsic semiconductors are further subdivided into two groups :

i) n-type semiconductor, ii) p-type semiconductor.

9.3.1 n-type semiconductor

A semiconductor doped with a pentavalent impurity is called n-type semiconductor. Typical examples of pentavalent impurities are arsenic (As, Z=33) and antimony (Sb, Z = 51). When these impurities are added to pure germanium crystal, four valence electrons of impurity atom form covalent bonds with the four valence electrons of the neighbouring Ge atoms. The fifth valence electron of impurity atom has no place to form the covalent bond and remains free to move randomly in the crystal lattice as shown in Fig.9.2(a). Thus each impurity atom donates a free electron to the semiconductor. Hence the impurity is called donor impurity. The semiconductor containing donor type impurity is called n-type semiconductor because it has negative charge carriers, i.e., electrons.



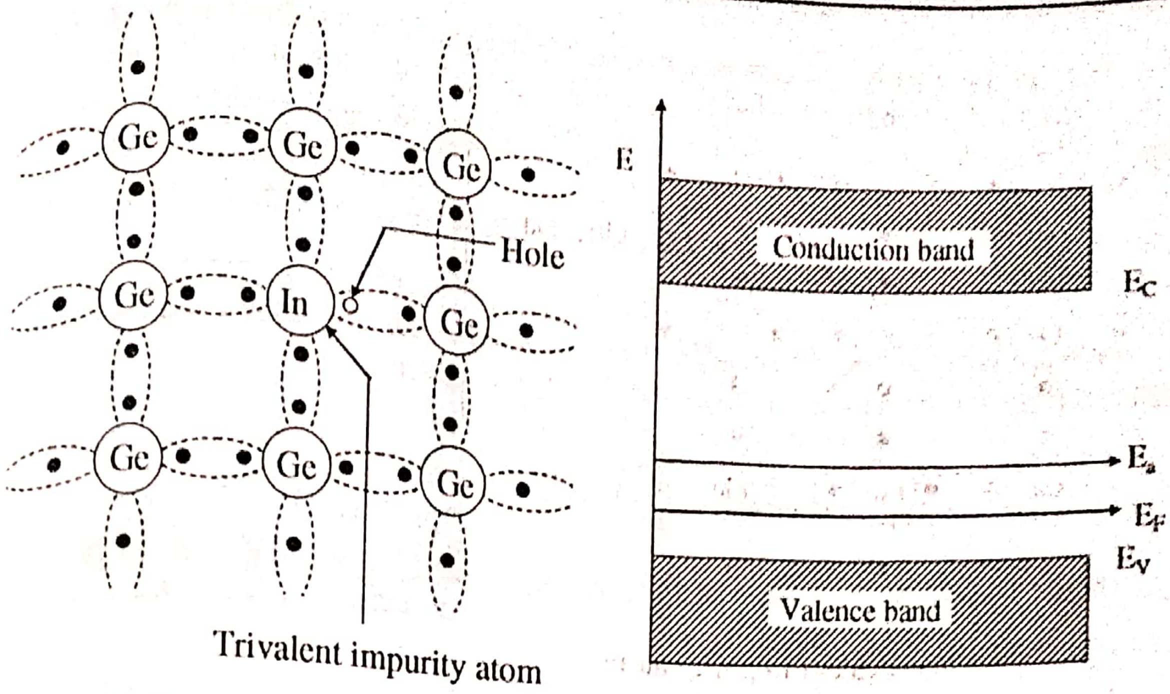
(a) Bond structure (b) Energy band diagram

Fig. 9.2 Bond structure and energy band diagram of n-type semiconductor

Although each arsenic atom donates one electron, yet doping the crystal with just a trace of impurity provides enough free electrons to move through the crystal structure and act as charge carrier. In the case of germanium, doped with donor impurities the energy required to move an electron from donor impurity to the conduction band is in the order of 0.0127 eV. The energy band diagram for n-type semiconductor is shown in Fig. 9.2(b). Here E_d represents the energy level corresponding to donor impurities and it lies just below the conduction band. It should be noted that in n-type semiconductors, the electrons are majority charge carriers but there are still present some thermally generated holes which are called the minority charge carriers.

9.3.2 p-type semiconductor

A semiconductor doped with a trivalent impurity is called p-type semiconductor. Typical examples of trivalent impurities are gallium (Ga, Z=31) and indium (In, Z=49). When a small amount of Indium is added to germanium crystal, three valence electrons of impurity atoms form covalent bonds with three valence electrons from three neighbouring germanium atoms. There is a deficiency of one electron to complete the fourth bond. This electron deficiency is called the hole and it behaves like a positively charged particle as shown in Fig. 9.3(a). Since there is a strong tendency of semiconductor crystal to form covalent bonds, a hole attracts one electron from a nearby covalent bond. Consequently a new hole is created at the originally occupied place of electron. This hole is again filled by another electron from a nearby bond giving rise to another hole and so on. Thus a hole moves freely throughout the crystal lattice.



(a) Bond structure

(b) Energy band diagram

Fig. 9.3 Bond structure and energy band diagram of p-type semiconductor

An impurity that produces holes inside a semiconductor crystal is called acceptor impurity because it accepts electrons from the nearby germanium atoms. The semiconductor containing acceptor type of impurity is called p-type semiconductor. because the charge carriers are positive holes. The energy band diagram for p-type semiconductor is as shown in Fig. 9.3(b). Here E_a represents the energy level corresponding to the accepter impurities. When an intrinsic semiconductor is doped with accepter impurity, the concentration of holes in the valence band will be more than the concentration of electrons in conduction band and the Fermi level shifts towards the valence band. The accepter level is just above the valence band. It should be noted that in p-type semiconductors, holes are majority charge carriers and electrons are the minority charge carriers.

9.4 Carrier Concentration in Intrinsic Semiconductor

In intrinsic semiconductors, the charge carriers are electrons in the conduction band and holes in the valence band. The concentration of electrons and holes in a semiconductor can be obtained from the knowledge of the densities of available states in the conduction band and the valence band as well as the Fermi-Dirac distribution function.

Concentration of electrons in the conduction band :

Let us assume the number of states in the energy range E and $E+dE$ is $Z(E)dE$, where $Z(E)$ is the density of states. Since each of these states has an occupation probability $F(E)$, the number of electrons in the energy range E and $E+dE$ is given by

$$dn = Z(E)\, F(E)\, dE \qquad \text{......} \quad (9.1)$$

If E_c is the energy corresponding to the bottom of the conduction band, the number of electrons in the conduction band can be calculated by integrating the above equation from E_c to the energy corresponding to the top of the conduction band ∞. Then,

$$n = \int_{E_c}^{\infty} Z(E)F(E)dE \qquad \text{......} \quad (9.2)$$

We know that the density of states i.e., the number of energy states per unit volume within the energy interval E and $E + dE$ is given by,

$$Z(E)dE = \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}dE \qquad \text{.......} \quad (9.3)$$

Since the electron is moving in a periodic potential, its mass has to be replaced by its effective mass m_e^*. Hence,

$$Z(E)dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}E^{1/2}dE$$

For semiconductors, E starts at the bottom of the conduction band E_c. Hence,

$$Z(E)dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_c)^{1/2}dE \qquad \text{.......} \quad (9.4)$$

The Probability occupation of an electron in an energy state E under thermal equilibrium is

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} \qquad \text{........} \quad (9.5)$$

where, k_B is Boltzmann constant and T is temperature in Kelvin. E_F is the energy of the Fermi level.

$$F(E) = \left[1 + exp\left(\frac{E - E_F}{k_B T}\right)\right]^{-1}$$

for all possible temperatures, $E - E_F >> k_B T$

Hence, $$F(E) \cong exp\left(-\frac{E - E_F}{k_B T}\right) \cong exp\left(\frac{E_F - E}{k_B T}\right) \qquad \text{.......} \quad (9.6)$$

Substituting eqns (9.4) and (9.6) in the eqn. (9.2), we get

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_c)^{1/2} exp\left(\frac{E_F - E}{k_B T}\right) dE$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} exp\left(\frac{E_F - E}{k_B T}\right) dE \qquad \text{...... (9.7)}$$

To solve this integral, let us put, $E - E_c = x$. If $E = E_c$, then $x = 0$

$\therefore E = E_c + x$ and $dE = dx$

$$\therefore\ n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} exp\left(\frac{E_F}{k_B T}\right) \int_0^{\infty} x^{1/2} exp \frac{-(E_c + x)}{k_B T} dx$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2} exp\left(\frac{E_F - E_c}{k_B T}\right) \int_0^{\infty} x^{1/2} exp\left(\frac{-x}{k_B T}\right) dx \qquad \text{....... (9.8)}$$

It can be shown that, using gamma function

$$\int_0^{\infty} x^{1/2} exp\left(\frac{-x}{k_B T}\right) dx = (k_B T)^{3/2} \frac{\pi^{1/2}}{2} \qquad \text{....... (9.8a)}$$

Hence, $$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} exp\left(\frac{E_F - E_c}{k_B T}\right)(k_B T)^{3/2} \frac{\pi^{1/2}}{2}$$

i.e., The number of electrons per unit volume of the material is given by

$$n = 2\left(\frac{2\pi\, m_e^* k_B T}{h^2}\right)^{3/2} exp\left(\frac{E_F - E_c}{k_B T}\right) \qquad \text{....... (9.9)}$$

This is the expression for concentration of electrons in the conduction band of an intrinsic semiconductor.

Concentration of holes in the valence band :

Let us assume that, the number of states in the energy range E and $E+dE$ is equal to $Z(E)dE$, where $Z(E)$ is the density of states. The probability that a hole occupies a level E in the valence band is equal to $(1-F(E))$. The number of holes in the energy range E and $E + dE$ is given by,

$$dp = Z(E)[1-F(E)]dE \qquad \text{....... (9.10)}$$

Since the energy range involved here is much lower than E_F, the probability occupation of hole is,

$$1-F(E)=1-\frac{1}{1+exp\left(\frac{E-E_F}{k_BT}\right)}$$

$$=1-\left[1+\left(exp\frac{E-E_F}{k_BT}\right)\right]^{-1}$$

Since, $E - E_F >> k_BT$

$$1-F(E)=exp\left(\frac{E-E_F}{k_BT}\right) \qquad \text{.......} \quad (9.11)$$

The density of states for the holes in the valence band is

$$Z(E)dE=\frac{4\pi}{h^3}(2m_h^*)^{3/2}E^{1/2}dE$$

Since E starts from the top

$$Z(E)dE=\frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_v-E)^{1/2}dE \qquad \text{......} \quad (9.12)$$

To calculate the number of holes in the valence band the eqn. (9.12) has to be integrated from $-\infty$ to the energy corresponding to the valence band E_v. Therefore, the number of holes in the valence band per unit volume is given by

$$p=\int_{-\infty}^{E_v}\frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_v-E)^{1/2}exp\left(\frac{E-E_F}{k_BT}\right)dE \qquad \text{.......} \quad (9.13)$$

$$p=\frac{4\pi}{h^3}(2m_h^*)^{3/2}exp\left(\frac{-E_F}{k_BT}\right)\int_{-\infty}^{E_v}(E_v-E)^{1/2}exp\left(\frac{E}{k_BT}\right)dE \qquad \text{.....} \quad (9.13a)$$

To solve this integral, let us take, $E_v - E = x$ (or) $E = E_v - x$ and $dE = -dx$

If $E = E_v$ then $x = 0$

$$\therefore \int_{-\infty}^{E_v}(E_v-E)^{1/2}exp\left(\frac{E}{k_BT}\right)dE=-\int_{\infty}^{0}x^{1/2}exp\frac{(E_v-x)}{k_BT}dx$$

$$=\exp\left(\frac{E_v}{k_BT}\right)\int_0^{\infty}x^{1/2}exp\left(\frac{-x}{k_BT}\right)dx$$

It can be shown that, using gamma function of eqn. (9.8a), we have

$$\int_{-\infty}^{E_v}(E_V-E)^{1/2}\,exp\left[\frac{E}{k_BT}\right]dE = exp\,\frac{E_v}{k_BT}(k_BT)^{3/2}\,\frac{\pi^{1/2}}{2} \qquad \text{........ (9.13b)}$$

Hence, from eqns (9.13a) and (9.13b) we have

$$p=\frac{4\pi}{h^3}(2m_h^*)^{3/2}exp\left(\frac{E_V-E_F}{k_BT}\right)(k_BT)^{3/2}\,\frac{\pi^{1/2}}{2}$$

i.e., The number of holes per unit volume of the material is given by

$$p=2\left(\frac{2\pi\, m_h^* k_BT}{h^2}\right)^{3/2} exp\left(\frac{E_v-E_F}{k_BT}\right) \qquad \text{........ (9.14)}$$

This is the expression for the number of holes in the valence band of an intrinsic semiconductor.

Intrinsic carrier concentration :

Since, in intrinsic semiconductor the number of electrons and holes are equal. Therefore, $n = p = n_i$ is called intrinsic carrier concentration. Hence,

$$n_i^2=np=4\left(\frac{2\pi k_BT}{h^2}\right)^3\left(m_e^*m_h^*\right)^{3/2}.\, exp\frac{(E_v-E_c)}{k_BT}$$

$$=4\left(\frac{2\pi k_BT}{h^2}\right)^3\left(m_e^*m_h^*\right)^{3/2} exp\left(\frac{-E_g}{k_BT}\right) \qquad \text{..... (9.15)}$$

where, $E_c - E_v = E_g$ is the forbidden energy gap. Therefore,

$$n_i=2\left(\frac{2\pi k_BT}{h^2}\right)^{3/2}\left(m_e^*m_h^*\right)^{3/4} \mathrm{exp}\left(\frac{-E_g}{2k_BT}\right) \qquad \text{..... (9.16)}$$

Fermi level in intrinsic semiconductor :

We know that in intrinsic semiconductor the density of electrons in the conduction band is equal to the density of holes in the valence band. Hence, $n = p$. Therefore, from eqns (9.9) and (9.14) we have

$$2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2} exp\left(\frac{E_F-E_c}{k_BT}\right)=2\left(\frac{2\pi\, m_h^* k_BT}{h^2}\right)^{3/2} exp\left(\frac{E_v-E_F}{k_BT}\right)$$

or $$(m_e^*)^{3/2} exp\left(\frac{E_F - E_c}{k_B T}\right) = (m_h^*)^{3/2} exp\left(\frac{E_v - E_F}{k_B T}\right)$$

$$exp\left(\frac{2E_F}{k_B T}\right) = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} \exp\left(\frac{E_v + E_c}{k_B T}\right)$$

Taking logarithms on both sides, we get

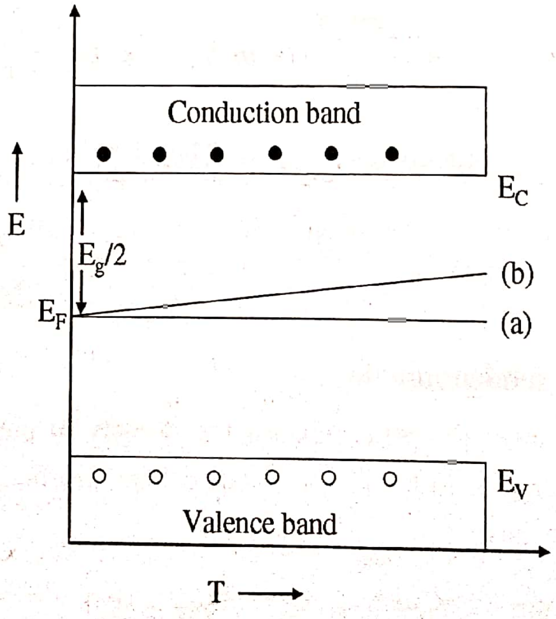
$$\frac{2E_F}{k_B T} = \frac{3}{2} log\left(\frac{m_h^*}{m_e^*}\right) + \left(\frac{E_v + E_c}{k_B T}\right)$$

i.e., $$E_F = \frac{3k_B T}{4} log\left(\frac{m_h^*}{m_e^*}\right) + \left(\frac{E_v + E_c}{2}\right) \quad ..... (9.17)$$

When $m_e^* = m_h^*$, $log\left(\frac{m_h^*}{m_e^*}\right) = log 1 = 0$, hence the eqn. (9.17) becomes,

$$E_F = \frac{E_v + E_c}{2}$$

Thus, the Fermi level lies mid way between the valence and conduction bands. The position of the Fermi level does not depend upon the temperature. Normally, $m_h^* > m_e^*$, so the Fermi level E_F is just above the middle of the energy gap and rises slightly with increasing temperature as shown in Fig. 9.4.



(a) Fermi level at T=0 K (b) Fermi level above T=0 K

Fig. 9.4 Position of Fermi level in an intrinsic semiconductor at various temperatures

9.5 Equation for Electrical Conductivity of Intrinsic Semiconductor

The electrical conductivity of a material depends on density of charge carriers and mobility of charge carriers. The drift velocity per unit electric field is called mobility. Therefore, the expression for electrical conductivity of a material is given by,

$$\sigma = ne\mu$$

where n is the density of charge carriers, e is the charge of the electron and μ is the mobility of charge carriers.

In semiconductors, there are two types of charge carriers, electrons and holes. Its conductivity is sum of the conductivities due to electrons and holes. Thus the electrical conductivity of a semiconductor is,

$$\sigma = \sigma_e + \sigma_h$$

Here, $\sigma_e = n_e e\mu_e$ is the conductivity due to electrons in the conduction band and

$\sigma_h = n_h e\mu_h$ is the conductivity due to holes in the valence band.

Therefore, the total conductivity of a semiconductor is,

$$\sigma = n_e e\mu_e + n_h e\mu_h \qquad \text{......} \quad (9.18)$$

In intrinsic semiconductor the density of electrons in the conduction band must be equal to the density of holes in the valence band and thus we can write, $n_e = n_h = n_i$. Therefore,

$$\sigma = n_i e(\mu_e + \mu_h) \qquad \text{.......} \quad (9.19)$$

where n_i is the intrinsic carrier concentration. We know that from eqn. (9.16)

$$n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} \left(m_e^* m_h^*\right)^{3/4} exp\left(\frac{-E_g}{2k_B T}\right)$$

where m_e^* is the effective mass of an electron, m_h^* is the effective mass of a hole and $E_g = E_c - E_v$ is the band gap. Putting the value of n_i in eqn. (9.19), we get,

$$\sigma = 2e(\mu_e + \mu_h)\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} \left(m_e^* m_h^*\right)^{3/4} exp\left(\frac{-E_g}{2k_B T}\right) \qquad \text{......} \quad (9.20)$$

Hence, the electrical conductivity depends upon the mobility of both holes and electrons and the forbidden energy gap between the valence and conduction bands. The mobility of charge carriers depends on temperature and is proportional to $T^{-3/2}$, i.e., $\mu \propto T^{-3/2}$.

$$\mu_e = \alpha\, T^{-3/2} \text{ and } \mu_h = \beta T^{-3/2}$$

Thus, $(\mu_e + \mu_h) = (\alpha + \beta)T^{-3/2} = \gamma T^{-3/2}$, where α, β are constants.

Therefore the electrical conductivity of an intrinsic semiconductor can be written as

$$\sigma = 2e\gamma T^{-3/2}\left(\frac{2\pi k_B}{h^2}\right)^{3/2} T^{3/2}\left(m_e^* m_h^*\right)^{3/4} exp\left(\frac{-E_g}{2k_B T}\right)$$

$$\sigma_i = A\exp\left(\frac{-E_g}{2k_B T}\right) \qquad \ldots\ldots (9.21)$$

Here, $A = 2e\gamma\left(\frac{2\pi k_B}{h^2}\right)^{3/2}\left(m_e^* m_h^*\right)^{3/4}$ is a constant.

Taking logarithm on both sides of eqn. (9.21),

$$ln(\sigma_i) = ln\, A + \left(\frac{-E_g}{2k_B T}\right)$$

A graph can be drawn between $ln\,(\sigma_i)$ verses $1/T$, the resulting graph is a straight line shown in Fig. 9.5. From the graph it is observed that the conductivity increases with increase of temperature. This graph has a negative slope equal to $(-E_g/2k_B)$.

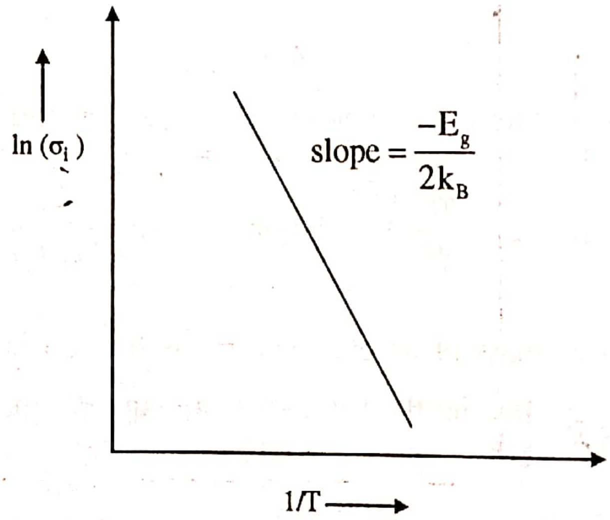


Fig. 9.5 Variation of $l_n\,(\sigma_i)$ with 1/T

Energy gap of a semiconductor :

The resistivity (ρ_i) can be written as (From eqn. 9.21),

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{A} exp\left(\frac{E_g}{2k_B T}\right)$$

$$\rho_i = B\exp\left(\frac{E_g}{2k_BT}\right) \qquad \text{........ (9.22)}$$

where, $B = 1/A$ = a constant.

Taking logarithm on both sides of eqn. (9.22)

$$ln(\rho_i) = ln\ B + \frac{E_g}{2k_BT} \qquad \text{....... (9.23)}$$

At different temperatures, the resistivity of an intrinsic semiconductor has been determined. We can measure energy gap E_g. A graph between $ln(\rho_i)$ verses (1/T) is shown in Fig. 9.6. The graph is a straight line and the slope of the line is equal to $(E_g/2k_B)$. Equating the slope of the graph to $(E_g/2k_B)$, the energy gap E_g can be determined. Therefore, from the graph,

$$\text{Slope} = \frac{dy}{dx} = \frac{E_g}{2k_B}$$

$$E_g = 2k_B\left(\frac{dy}{dx}\right) \qquad \text{...... (9.23a)}$$

where k_B is the Boltzmann constant.

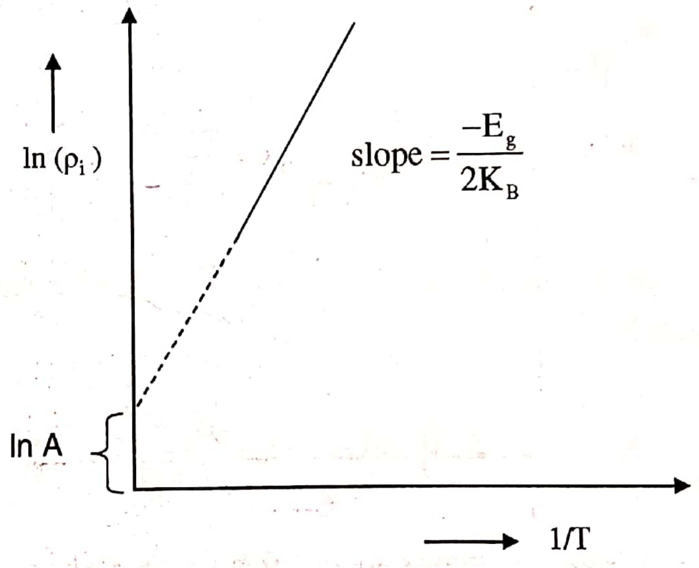


Fig. 9.6 Variation of ln (ρ_i) with (1/T)

9.6 Carrier Concentration in *n*-type Semiconductor

In *n*-type semiconductors the donor level just below the conduction band is shown in Fig. 9.7. Let E_d represents the energy of donor level and N_d is the donor concentration i.e., the number of donor atoms per unit volume of the material. Since the donor level is very nearer to the conduction band, the effect of valence band may be neglected. Therefore, we can say that the concentration of electrons in the conduction band must be equal to the number of positive ionized donors per unit volume in the donor level. The density of electrons in conduction band is given by the equation

$$n = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} exp\left(\frac{E_F - E_c}{k_B T}\right) \qquad \text{........ (9.24)}$$

where E_F is the energy of the Fermi level and E_c is the energy corresponding to the bottom of the conduction band. Therefore, the number of vacancies per unit volume in the donor level is,

$$N_d\left[1-F(E_d)\right] = N_d\left[1-\frac{1}{1+exp\left(\frac{E_d - E_F}{k_B T}\right)}\right]$$

$$= N_d\left[\frac{exp\left(\frac{E_d - E_F}{k_B T}\right)}{1+exp\left(\frac{E_d - E_F}{k_B T}\right)}\right]$$

$$= \frac{N_d}{1+exp\left(\frac{E_F - E_d}{k_B T}\right)}$$

If we assume that $E_F >> k_B T$ above the donor level then the density of positive ionized donors is given by

$$N_d\left[1-F(E_d)\right] \cong N_d\ exp\left(\frac{E_d - E_F}{k_B T}\right) \qquad \text{...... (9.25)}$$

At very low temperatures, when electron-hole pairs are not generated due to breaking of covalent bonds, the number of electrons in the conduction band must be equal to the number of ionized donors. i.e.,

$$2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} exp\left(\frac{E_F - E_c}{k_B T}\right) = N_d exp\left(\frac{E_d - E_F}{k_B T}\right)$$

Taking logarithm and rearranging we get,

$$\left(\frac{E_F - E_c}{k_BT}\right) - \left(\frac{E_d - E_F}{k_BT}\right) = logN_d - log2\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2}$$

$$2E_F - (E_d + E_c) = k_BT\log\frac{N_d}{2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2}}$$

$$E_F = \left(\frac{E_d + E_c}{2}\right) + \frac{k_BT}{2}log\frac{N_d}{2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2}} \qquad \text{......} \ (9.26)$$

Substituting E_F from eqn. (9.26) in eqn. (9.24) and rearranging, we get

$$n = 2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2} exp\left\{\frac{\left(\frac{E_d + Ec}{2}\right) + \frac{K_BT}{2}log\frac{N_d}{2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2}} - E_c}{k_BT}\right\}$$

$$= 2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2} exp\left\{\left(\frac{E_d - E_c}{2k_BT}\right) + \frac{1}{2}log\frac{N_d}{2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2}}\right\}$$

$$= 2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2} exp\left\{\left(\frac{E_d - E_c}{2k_BT}\right) + log\frac{N_d^{1/2}}{\left[2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2}\right]^{1/2}}\right\} \left[\because \frac{1}{2}\log a = \log a^{1/2}\right]$$

$$= 2\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/2}\left[exp\left(\frac{E_d - E_c}{2k_BT}\right)\right]\left(\frac{N_d}{2}\right)^{1/2}\frac{1}{\left(\frac{2\pi\, m_e^* k_BT}{h^2}\right)^{3/4}}$$

$$\left[\because exp(a+b) = exp(a).exp\ (b)\right]$$

$$= 2^{1/2}N_d^{1/2}\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2}\frac{1}{\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/4}}\, exp\left(\frac{E_d - E_c}{2k_BT}\right)$$

$$n = (2N_d)^{1/2}\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/4} exp\left(\frac{E_d - E_c}{2k_B T}\right) \quad \text{.......} \quad (9.27)$$

This is the expression for the carrier concentration in n-type semiconductor. Thus we find that the concentration of electrons in the conduction band is proportional to the square root of the donor concentration at moderately low temperatures.

From eqn. (9.26), at $T = 0$ K,

$$E_F = \frac{E_d + E_c}{2}$$

i.e., at 0 K, Fermi level lies exactly at the middle of the donor level E_d and the bottom of the conduction band E_c as shown in Fig. 9.7.

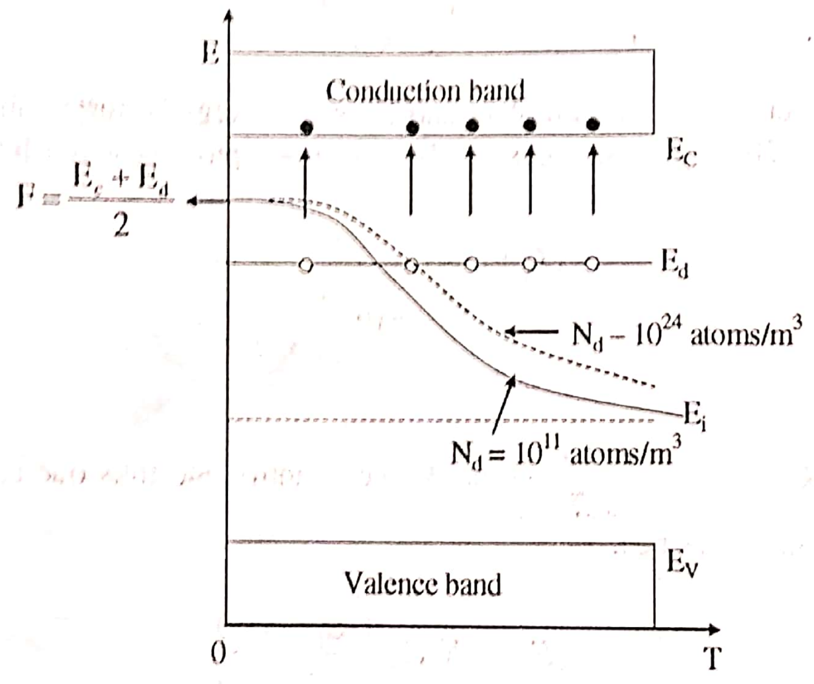


Fig. 9.7 Position of Fermi level with temperature in n-type semiconductor

Position of Fermi level with temperature :

When the temperature is increased, the Fermi level falls below the donor level and at high temperature the value of E_F becomes $\left(\frac{E_c + E_v}{2}\right)$. That is the Fermi level gradually moves towards the intrinsic Fermi level E_i as shown in Fig. 9.7. Therefore, at high temperatures the n-type semiconductor behaves as an intrinsic semiconductor.

9.7 Carrier Concentration in p-type Semiconductors

In p-type semiconductor the acceptor level is just above the valence band as shown in Fig.9.8. Let E_a represent the energy of acceptor level and N_a is the acceptor concentration i.e., the number of acceptor atoms per unit volume of the material. Since the acceptor level is very nearer to the valence band, the effect of conduction band may be neglected. So, we can say that the concentration of holes in the valence band must be equal to the number of vacancies per unit volume in the acceptor level. At very low temperatures all the acceptor levels are empty. With increase of temperatures acceptor atoms get ionized i.e., the electrons move from valence band and occupy the vacant sites in the acceptor energy level thereby leaving holes in the valence band. The density of holes in the valence band is given by eqn. (9.14).

$$p = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} exp\left(\frac{E_v - E_F}{k_B T}\right) \qquad \text{......} \quad (9.28)$$

where E_F is the energy of the Fermi level and E_v is the energy corresponding to the top of the valence band. Therefore, the density of ionized acceptors is given by,

$$N_a F(E_a) = \frac{N_a}{1 + exp\left(\frac{E_a - E_F}{k_B T}\right)}$$

Since $E_a - E_F >> k_B T$, $exp\left(\frac{E_a - E_F}{k_B T}\right)$ is a large quantity and thus one can be neglected from the denominator. Hence,

$$N_a F(E_a) = N_a exp\left(\frac{E_F - E_a}{k_B T}\right) \qquad \text{........} \quad (9.29)$$

Since the density of holes in the valence band is equal to the density of negative ionize acceptors,

i.e.,

$$2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} exp\left(\frac{E_v - E_F}{k_B T}\right) = N_a exp\left(\frac{E_F - E_a}{k_B T}\right)$$

$$\exp\left(\frac{E_v + E_a - 2E_F}{k_B T}\right) = \frac{N_a}{2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2}}$$

Taking logarithm on both sides we get,

$$\left(\frac{E_v + E_a - 2E_F}{k_BT}\right) = log\frac{N_a}{2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}}$$

$$E_F = \left(\frac{E_v + E_a}{2}\right) - \frac{k_BT}{2} log\frac{N_a}{2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}} \quad \ldots\ldots (9.30)$$

Substituting E_F from eqn. (9.30) in eqn. (9.28) and rearranging, we get

$$p = 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} exp\left\{\frac{E_v - \frac{E_v + Ea}{2} + \frac{k_BT}{2}\log\frac{N_a}{2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}}}{k_BT}\right\}$$

$$= 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} exp\left[\frac{E_v - E_a}{2k_BT} + \frac{1}{2} log\frac{N_a}{2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}}\right]$$

$$= 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} exp\left\{\frac{E_v - E_a}{2k_BT} + log\frac{N_a^{1/2}}{\left[2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}\right]^{1/2}}\right\} \quad \left(\because \frac{1}{2}\log a = \log a^{1/2}\right)$$

$$= 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}\left[\exp\left(\frac{E_v - E_a}{2k_BT}\right)\right]\frac{N_a^{1/2}}{\left[2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}\right]^{1/2}} \quad \left[\because exp(a+b) = exp(a).exp(b)\right]$$

$$= 2\left(\frac{2\pi\, m_h^* k_B T}{h^2}\right)^{3/2} exp\left(\frac{E_v - E_a}{2k_B T}\right)\left(\frac{N_a}{2}\right)^{1/2} \frac{1}{\left(\frac{2\pi\, m_h^* k_B T}{h^2}\right)^{3/4}}$$

$$= 2^{1/2} N_a{}^{1/2}\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/4} \exp\left(\frac{E_v - E_a}{2k_B T}\right)$$

$$p = (2N_a)^{1/2}\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/4} \exp\left(\frac{E_v - E_a)}{2k_B T}\right) \qquad \text{......} \quad (9.31)$$

This is the expression for the carrier concentration in *p*-type semiconductor. Thus find that the concentration of holes in the valence band is proportional to the square ot of the acceptor concentration at moderately low temperatures.

From equation (9.30), at T = 0 K,

$$E_F = \frac{E_a + E_v}{2}$$

i.e., at 0 K, Fermi level lies exactly at the middle of the acceptor level E_a and above valence band E_v as shown in Fig. 9.8.

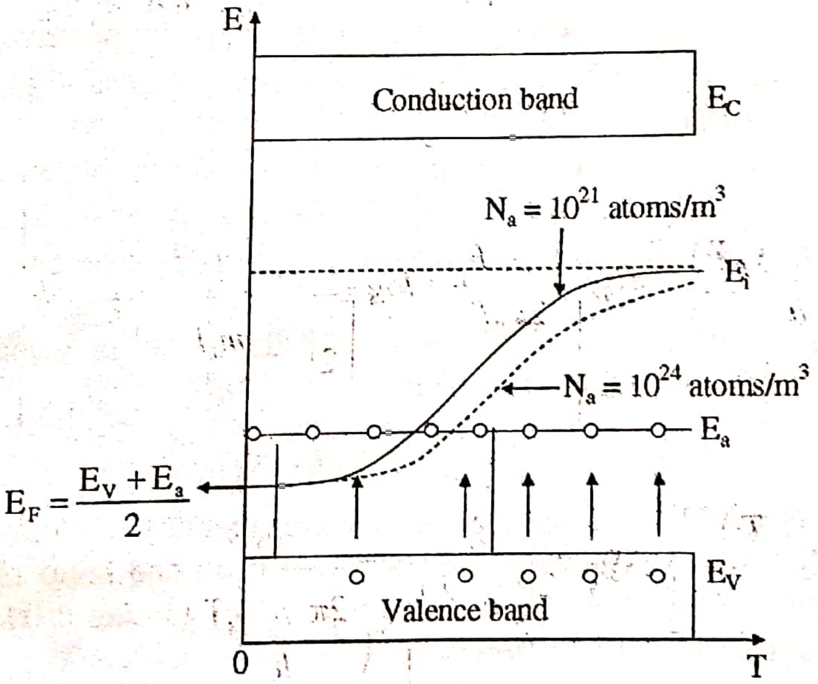


Fig. 9.8 Position of Fermi level with temperature in P-type semiconductor

Position of Fermi level with temperature :

When the temperature is increased, the Fermi level rises. At high temperatures the value of E_F becomes $\left(\frac{E_c + E_v}{2}\right)$. That is the Fermi level gradually moves towards the intrinsic Fermi level E_i as shown in Fig. 9.8. Therefore, at high temperatures the p-type semiconductor behaves as an intrinsic semiconductor.

9.8 Drift and Diffusion Currents

9.8.1 Drift current

We know that, the electric current arises due to the movement of charge carriers in the material. In semiconductors, the charge carriers are electrons in the conduction band and holes in the valence band. Due to thermal agitation, they move randomly in all directions and hence the net current in any direction is zero. When an electric field E is applied across a semiconductor, every charge carrier experiences a force due to electric field and drifts in the direction of the force. Thus, a charge carrier acquires an average velocity which is called the drift velocity and it gives rise to the *drift current. The total current due to holes and electrons in the presence of applied electric field is called the drift current.*

The electric current density is given by

$$J = nev_d$$

where v_d is the drift velocity. Also, $J = \sigma E$, where σ is the electrical conductivity of the material. In steady state the electrons are moving towards the positive terminal of the potential with drift velocity v_d m/s. The drift velocity is proportional to the applied electric field intensity E. Therefore,

$$v_d \propto E$$

$$v_d = \mu E \qquad \text{....... (9.32)}$$

Here 'μ' is called mobility of electrons and it defines as the velocity gained by an electron per unit electric field intensity. Thus,

$$J = nev_d = ne\mu E$$

Due to the current flow in semiconductors, the mobility of electrons and holes are generally different. Thus, the drifting motion of electrons and holes in the presence of applied electric field causes the drift current. The corresponding drift current densities due to electrons and holes are

$$J_n(\text{drift}) = ne\mu_e E \qquad \text{....... (9.33)}$$

$$J_p(\text{drift}) = pe\mu_h E \qquad \text{.......} \ (9.34)$$

Therefore, the total drift current density, $J_{\text{drift}} = J_n(\text{drift}) + J_p(\text{drift})$

$$J_{\text{drift}} = ne\mu_e E + pe\mu_h E$$
$$= eE[n\mu_e + p\mu_h] \qquad \text{......} \ (9.35)$$

In case of intrinsic semiconductor $n = p = n_i$, Hence,

$$J_{\text{drift}} = n_i \, e \, E \, [\mu_e + \mu_h] \qquad \text{.......} \ (9.36)$$

Since, $J = \sigma E$, the electrical conductivity of an intrinsic semiconductor due to drift action of electrons and holes is given by $\sigma_{\text{drift}} = n_i \, e \, [\mu_e + \mu_h]$.

9.8.2 Diffusion current

In addition to drift current there is also another current called diffusion current. The diffusion current is due to the flow of charge carriers when there is non-uniform concentration of charge carriers in semiconductors. The excess carriers move from the region of higher density to the region of lower density tending to produce a uniform distribution. The motion of charge carriers from region of higher concentration to the region of lower concentration leads to a current called diffusion current. In general the concentration of carriers varies with distance in a semiconductor, this constitutes a concentration gradient. The diffusion current is proportional to the concentration gradient.

Let us suppose that the concentration *dn* of electrons varies with distance *x* in the semiconductor, the concentration gradient being *dn/dx*.

According to Fick's law the rate at which the carriers diffuse is proportional to the density gradient and the movement is in the direction of negative gradient. The rate of flow of electrons is proportional to – *dn/dx* from which the rate of flow across unit area is equal to $-D_n \, dn/dx$, where D_n is called the diffusion coefficient for electrons in the semiconductor. This flow constitutes an electron current density and since conventional current is the rate of positive charge, $J_n(\text{diffusion}) = -e(\text{rate of flow across unit area})$

$$= eD_n \frac{dn}{dx} \qquad \text{........} \ (9.37)$$

Similarly, if an excess hole concentration is created in the region, hole diffusion takes place; the corresponding expression for holes in a semiconductor is

$$J_p(\text{diffusion}) = -eD_p \frac{dp}{dx} \qquad \text{.......} \ (9.38)$$

where, D_p is the diffusion constant for holes. For the same type of gradient the hole and electron currents are in opposite directions owing to the opposite signs of their electrical charge. Hence total diffusion current density,

$$J_{diffusion} = J_n(\text{diffusion}) + J_p(\text{diffusion})$$

$$J_{diffusion} = -eD_p \frac{dp}{dx} + eD_n \frac{dn}{dx} \qquad \text{......} \quad (9.39)$$

Thus in a semiconductor, the total current density is due to drift of carriers and diffusion of carriers. *Hence the total current flow in a semiconductor* in one dimension is expressed as

$$J = J_{drift} + J_{diffusion}$$

$$J = eE\,[n\mu_e + p\mu_h] - eD_p \frac{dp}{dx} + eD_n \frac{dn}{dx}$$

$$J = e\left[\left(n\mu_e E + D_n \frac{dn}{dx}\right) + \left(p\mu_h E - D_p \frac{dp}{dx}\right)\right] \qquad \text{......} \quad (9.40)$$

9.9 Einstein's Equation

The mobility μ expresses the ability of carriers to drift while the diffusion constant 'D' expresses the ability of carriers to diffuse. The concept of mobility derives from the net carrier motion resulting from random collisions of these charges with the lattice atoms under the action of applied electric field. Similarly, the diffusion process can be described in terms of a net motion of carriers superimposed on their random thermal motion, under the effect of a concentration gradient and involving collisions with the lattice. Einstein showed that the parameters describing the drift and diffusion processes, mobility and diffusion coefficient are directly related. Under equilibrium conditions, the drift and diffusion currents due to an excess density of electrons, from eqns (9.33) and (9.37) are equal. Therefore,

$$(dn)eE\mu_e = eD_n \frac{dn}{dx}$$

The force F on excess carriers restoring equilibrium is given by the product of excess charge and electric field

$$F = (dn)eE = (eD_n / \mu_e) \frac{dn}{dx}$$

This force also depends on the thermal energy of excess carriers. According to the kinetic theory of gases, if the molecular concentration of a gas increases by (dn), the gas pressure increases by $(dn)k_BT$. The force corresponding to the pressure gradient is given by $F = k_BT\,\frac{dn}{dx}$. By making an analogy between the excess carriers in semiconductors and gas molecules in a container, we have

$$k_BT\,\frac{dn}{dx} = (eD_n / \mu_e)\,\frac{dn}{dx}$$

$$k_BT = eD_n / \mu_e$$

i.e.,

$$D_n = \frac{\mu_e k_BT}{e} \text{ (or) } \frac{D_n}{\mu_e} = \frac{k_BT}{e} \quad \text{.........} (9.41)$$

This is the relation between diffusion coefficient and mobility of charge carriers and is termed as *Einstein relation*. This relation for holes is given by

$$D_p = \frac{\mu_h k_BT}{e} \text{ (or) } \frac{D_p}{\mu_h} = \frac{k_BT}{e} \quad \text{......} (9.42)$$

$$\therefore \quad \frac{D_n}{D_p} = \frac{\mu_e}{\mu_h} \text{ (or) } \frac{D_n}{\mu_e} = \frac{D_p}{\mu_h} = \frac{k_BT}{e} \quad \text{......} (9.43)$$

For silicon D_n=3.4 × 10^{-3} m^2/s and D_p=1.3 × 10^{-3} m^2/s, for germanium D_n = 9.9 × 10^{-3} m^2/ s and D_p= 47 × 10^{-3} m^2/s.

According to Einstein equation, the ratio between the diffusion constant and the mobility of charge carriers is directly proportional to the absolute temperature of the semiconductor. Further, the Einstein equation tells us that when the temperature is increased both the diffusion and mobility of charge carriers increased.

9.10 Hall Effect

When a slab of metal or semiconductor carrying current is placed in a transverse magnetic field, a potential difference is produced in the direction normal to both current and magnetic field. This phenomenon is called Hall effect and the generated voltage is known as Hall voltage. It was discovered by E.H. Hall in 1879.

Consider a slab of conductor in which a current I is flowing in the positive X-direction as shown in Fig. 9.9. Let a magnetic field B be applied along the Z-direction then the electrons experience a Lorentz force given by,

$$F_L = -Bev_d \quad \text{.......} (9.44)$$

where e is charge of the electron and v_d is the drift velocity of the electron.

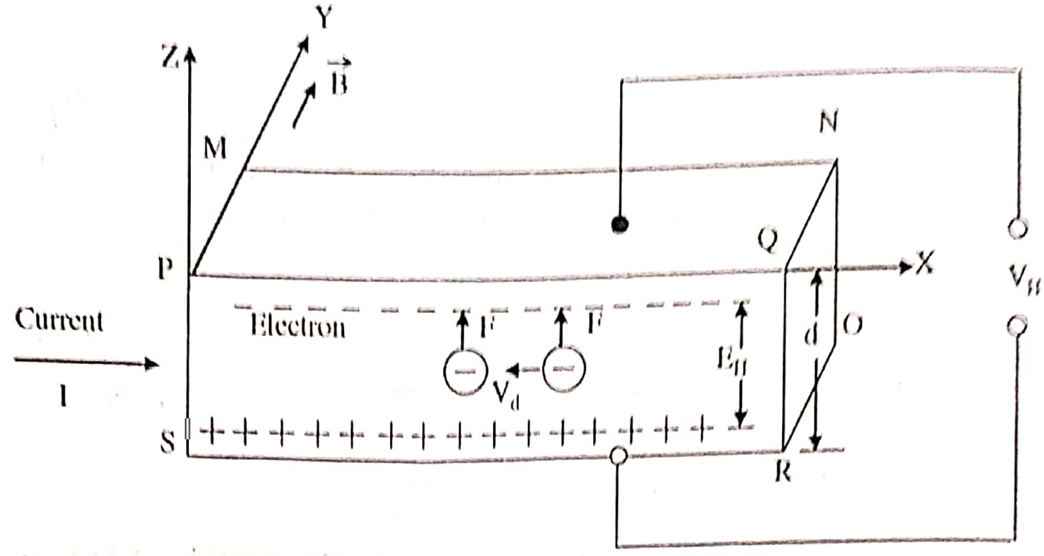


Fig. 9.9 Hall effect

Applying the Fleming's left hand rule, the force exerted on the electrons is in the negative y-direction. Therefore, the electrons are deflected in the downward direction. As a result, the density of electrons increases in the lower end of the material due to which its bottom surface becomes negatively charged. On the other hand, the loss of electrons from the upper end causes the top edge of the material to become positively charged. Hence, potential V_H called Hall voltage appears between the upper and lower surfaces of the semiconductor which establishes an electric field E_H called the Hall electric field. The electric field E_H exerts an upward force F_H in the electron is given by,

$$F_H = -eE_H \qquad \text{.......} \quad (9.45)$$

Now, as the deflection of electrons continues in the downward direction due to the Lorentz force, it also contributes to the growth of the Hall electric field. As a result, the force F_H, which acts on the electron in the upward direction, also increases. At equilibrium position two forces are equal, $F_L = F_H$

$$eE_H = eBv_d \quad \text{(or)} \quad E_H = Bv_d$$

For *n*-type material the charge carriers are electrons, and the current density J is,

$$J = -nev_d$$

where n is the concentration of charge carriers.

$$\text{Therefore,} \qquad v_d = -\frac{J}{ne} \quad \text{(or)} \quad E_H = -\frac{JB}{ne}$$

The Hall effect is described in terms of the Hall coefficient R_H and is given by

$$R_H = -\frac{1}{ne}$$

$$\text{Hence,} \qquad E_H = R_H JB$$

i.e.,
$$R_H = \frac{E_H}{JB} = -\frac{1}{ne} \qquad \text{...... (9.46)}$$

The Hall coefficient can be evaluated by substituting the quantities E_H, J and B. By knowing the Hall coefficient the carrier density n can be estimated. Since, the charge carriers are holes for p-type material. The Hall coefficient is,

$$R_H = \frac{E_H}{JB} = \frac{1}{pe} \qquad \text{....... (9.47)}$$

where p is the density of holes. The Hall coefficient R_H is inversely proportional to density of charge carriers.

Determination of Hall coefficient :

The Hall electric field per unit current density per unit magnetic induction is called Hall coefficient (R_H). If 'w' is the width of the sample across which Hall voltage V_H is measured

$$E_H = \frac{V_H}{w}$$

Therefore,
$$R_H = \frac{E_H}{JB} = \frac{V_H}{JBw}$$

$$V_H = R_H JBw$$

If t is the thickness of the sample, then its cross section is wt and current density

$$J = \frac{I}{wt}$$

Hence,
$$V_H = \frac{R_H IB}{t}$$

$\therefore$
$$R_H = \frac{V_H t}{IB} \qquad \text{....... (9.48)}$$

The Hall voltage V_H will be opposite for n–type and p–type semiconductors.

Applications of Hall effect :

The Hall effect measurements are used to ascertain the following information about the solid.

i) The sign of charge carriers can be determined.

ii) The carrier density can be estimated.

iii) The mobility of charge carriers can be measured directly.

iv) It can be used to determine whether the given material is a metal, an insulator or a semi-conductor.

v) The magnetic field can be measured by knowing the values of Hall voltage and Hall coefficient.